IEEE Distinguished Lecture 2015

Welcome to *kQ* Theory for Wireless Power Transfer

Takashi Ohira, *Fellow*, *IEEE*

Director, Future Vehicle City Research Center

Toyohashi University of Technology



Brief Review on WPT History

People in 20th century believed WPT emanates from coupling coefficient k between coils.

Drastically altered by an experiment at MIT in 2007.
Works even in low k, for a longer distance [1].
The key to their success was not k alone but k times Q.

Recognize kQ as essence of WPT. You may then ask:

Q1 Does kQ exist in every kind of coupling?
Q2 Can we derive kQ from circuit parameters?
Q3 How does kQ relate to maximum efficiency?

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[1] A. Kurs, et. al., "Wireless power transfer via strongly coupled magnetic resonators," *Science*, vol. 317, pp.83-86, July 2007.

This Lecture Clarifies …

1. What is kQ
2. kQ for inductive coupling
3. kQ for capacitive coupling
4. kQ for arbitrary scheme coupling
5. How to estimate η_{max} from kQ

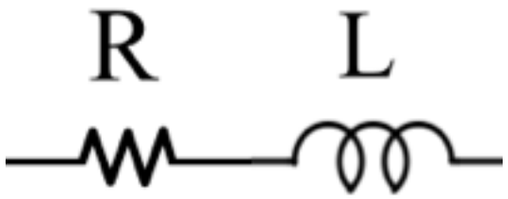
This Lecture Clarifies …

1. **What is *kQ***
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What is *kQ*

- ***kQ* is great news to power transfer engineers**

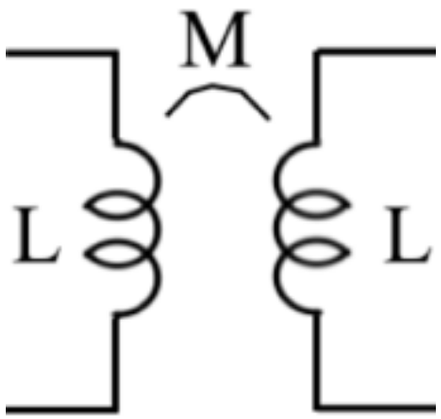
cheerful wedding



$$Q = \frac{\omega L}{R}$$

Quality Factor





$$k = \frac{M}{L}$$

Coupling Coefficient

- ***kQ* works as an essential figure of merit in place of usual S_{21}**
- ***kQ* keeps invariant to terminal conditions**

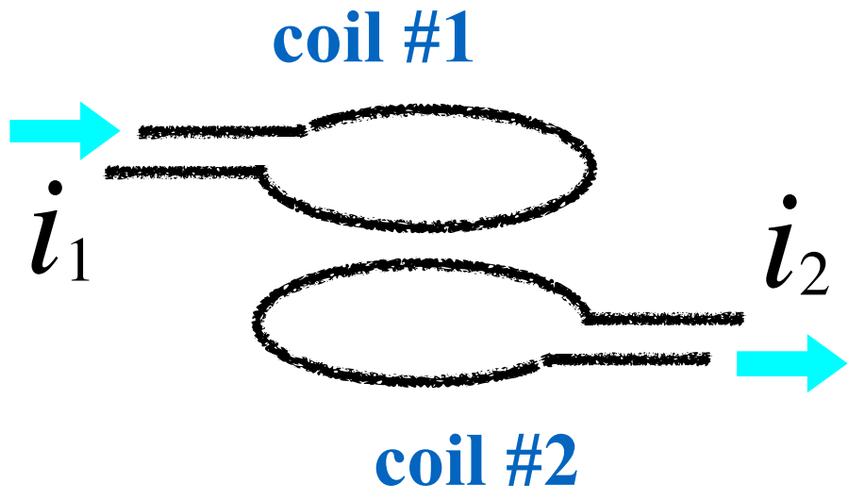
Figures of Merit at a Glance

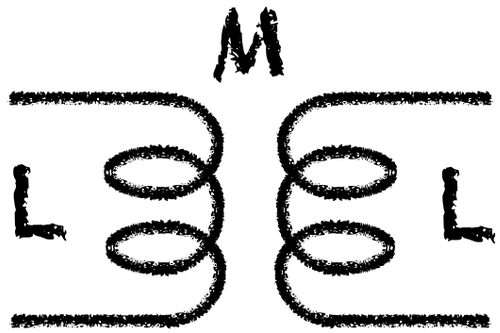
FoM	pros	cons
S_{21}	ready to measure by VNA	valid only for specified Z_o say 50Ω
k	easy to imagine by physics	definition ambiguous in general two-port
kQ	defined in any WPT works even for unknown Z_o	need to learn the formula before use

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kQ for inductive coupling





$$k = \frac{M}{L} \qquad Q = \frac{\omega L}{R}$$

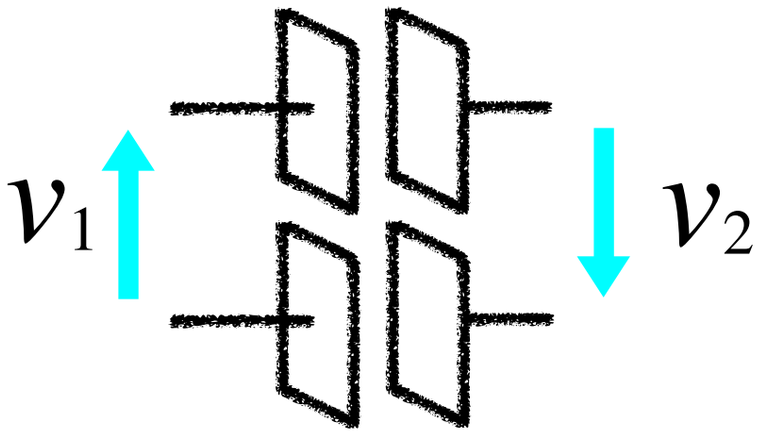
$$\therefore \quad kQ = \frac{M}{L}\frac{\omega L}{R} = \frac{\omega M}{R}$$

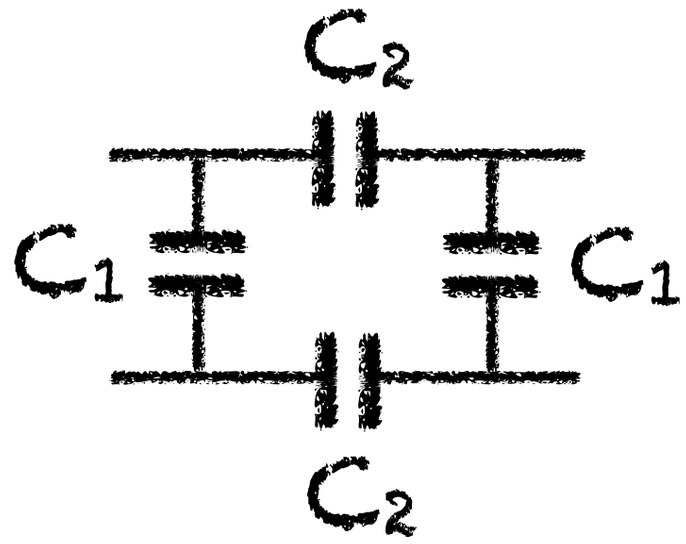
Note: k and Q are conventionally defined

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kQ for capacitive coupling



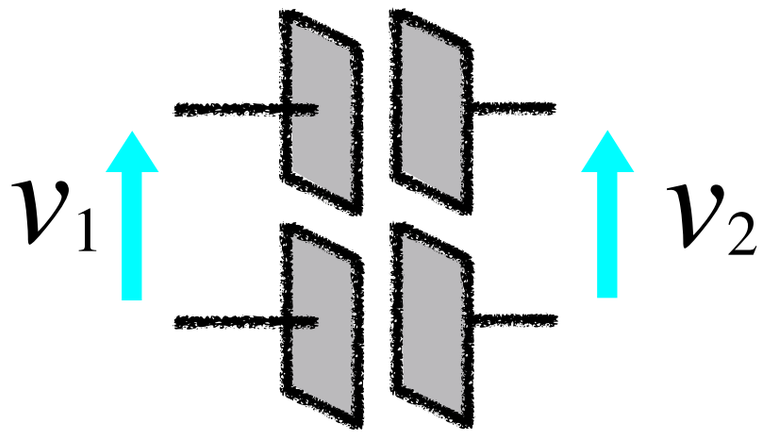
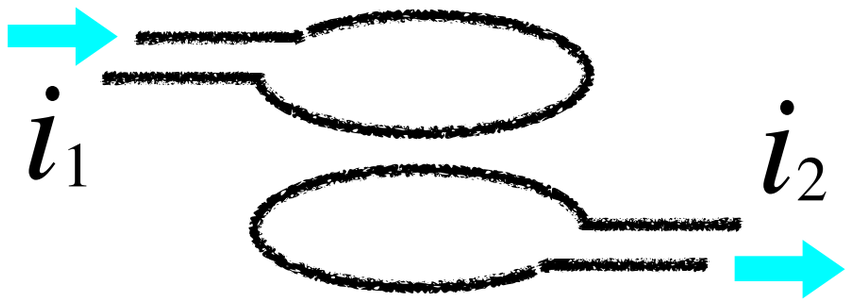
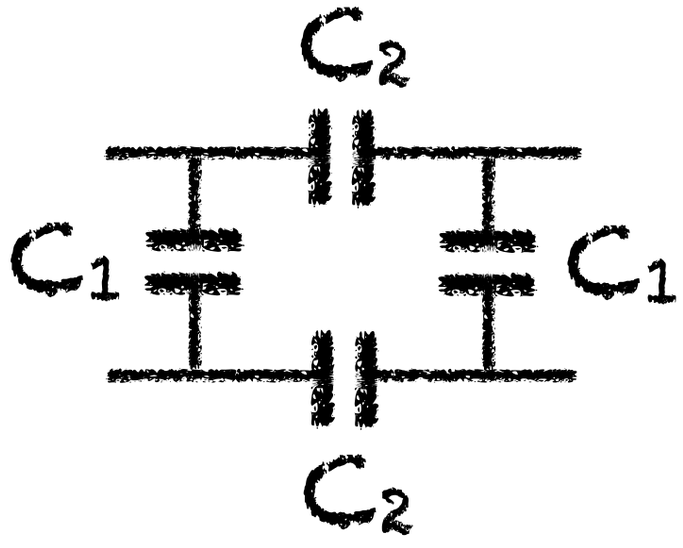
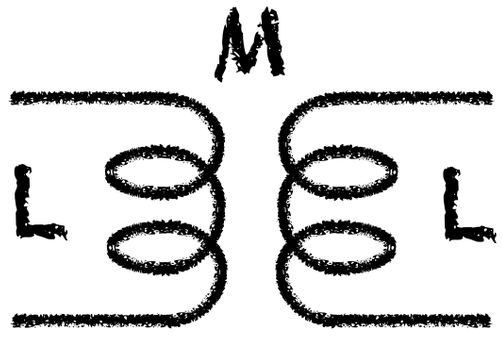


$$k = \frac{C_2}{2C_1 + C_2} \qquad Q = \omega\left(C_1 + \frac{C_2}{2}\right)R$$

$$\therefore\ kQ = \frac{C_2}{2C_1 + C_2}\omega\left(C_1 + \frac{C_2}{2}\right)R = \frac{1}{2}\omega C_2 R$$

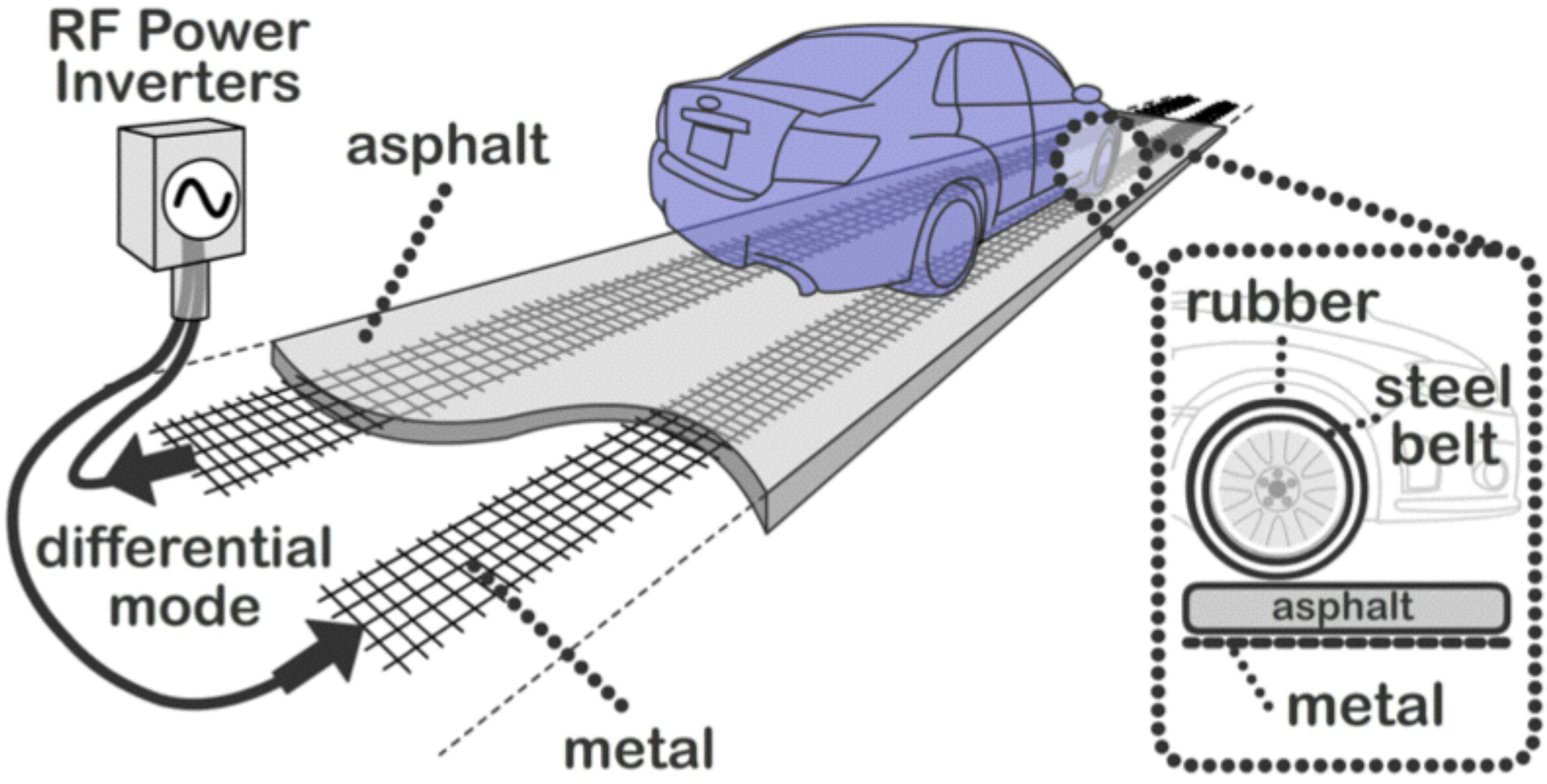
Note: *k* and *Q* are conventionally defined

kQ formulas for typical schemes

coupling	capacitive	inductive
struc-ture		
equi-valent		
kQ	$\frac{1}{2}\omega C_2 R$	$\frac{\omega M}{R}$

But in practical systems…

so complicated as



How can we calculate kQ ?

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kQ for arbitrary scheme coupling

$$\mathbf{Z}=\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}=\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}+j\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}$$



$$\mathbf{Z}=50(\mathbf{I}+\mathbf{S})(\mathbf{I}-\mathbf{S})^{-1}$$

$$|Z_{21}|=\sqrt{R_{21}^2+X_{21}^2}$$

$$|\mathbf{R}|=\begin{vmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{vmatrix}=R_{11}R_{22}-R_{12}R_{21}$$

General *kQ* formula

$$kQ=\frac{|Z_{21}|}{\sqrt{|\mathbf{R}|}}$$

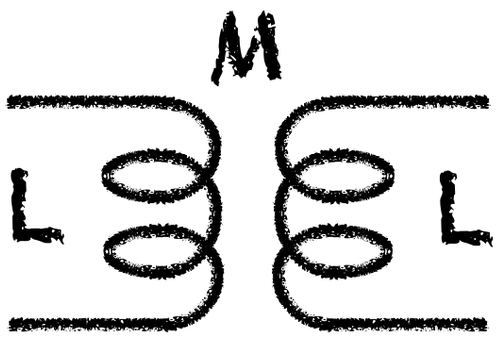
transfer function

loss factor

[2] T. Ohira, "Maximum available efficiency formulation based on a black-box model of linear two-port power transfer systems," *IEICE Electronics Express*, vol. 11, no. 13, July 2014.

Apply to inductive coupling

$$\mathbf{Z} = \begin{bmatrix} R + j\omega L & j\omega M \\ j\omega M & R + j\omega L \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} + j\omega \begin{bmatrix} L & M \\ M & L \end{bmatrix}$$



$$|z_{21}| = \omega M \qquad |\mathbf{R}| = \begin{vmatrix} R & 0 \\ 0 & R \end{vmatrix} = R^2$$

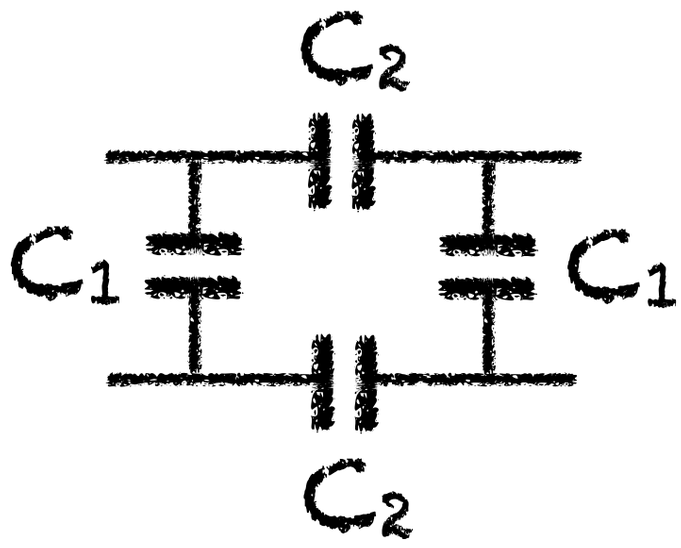
$$\therefore \quad kQ = \frac{|z_{21}|}{\sqrt{|\mathbf{R}|}} = \frac{\omega M}{R}$$

Consistent with the conventional *kQ*

Apply to capacitive coupling

$$\mathbf{Y} = \left(\frac{1}{R} + j\omega C_1\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{2} j\omega C_2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$y_{21} = -\frac{1}{2} j\omega C_2$$



$$|\mathbf{G}| = \begin{vmatrix} \frac{1}{R} & 0 \\ 0 & \frac{1}{R} \end{vmatrix} = \frac{1}{R^2}$$

$$\Rightarrow \quad kQ = \frac{|y_{21}|}{\sqrt{|\mathbf{G}|}} = \frac{1}{2}\omega C_2 R$$

Consistent with the conventional *kQ* again

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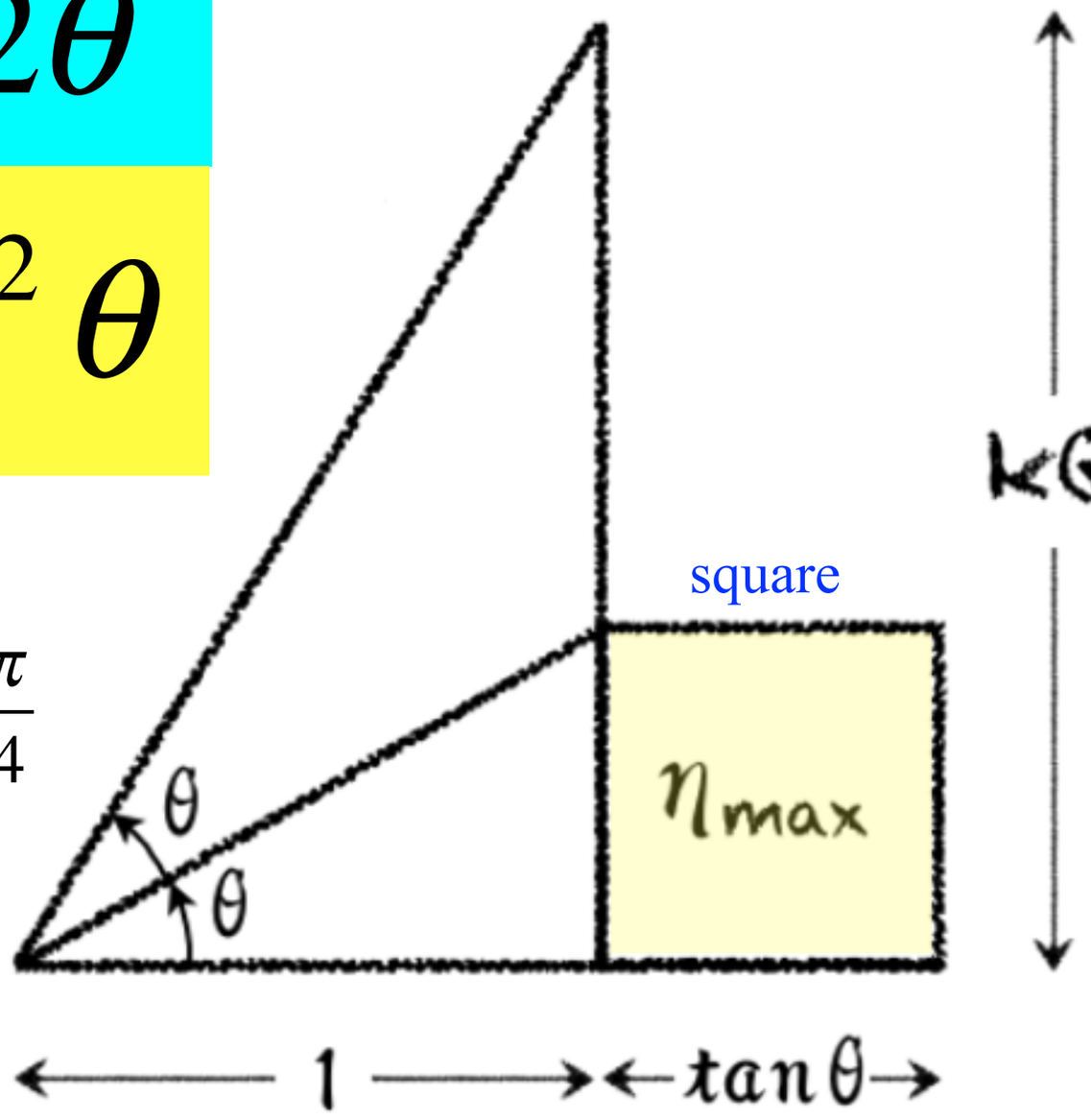
How to estimate η_{max} from kQ

elegant relationship

$$kQ = \tan 2\theta$$

$$\eta_{\max} = \tan^2 \theta$$

$$0 \le \theta \le \frac{\pi}{4}$$



"efficiency tangent"

[3] T. Ohira, "Angular expression of maximum power transfer efficiency in reciprocal two-port systems," *IEEE Wireless Power Transfer Conference*, pp.228-230, Jeju, May 2014.

Estimate kQ and η_{max} from $\mathbf{Z}$



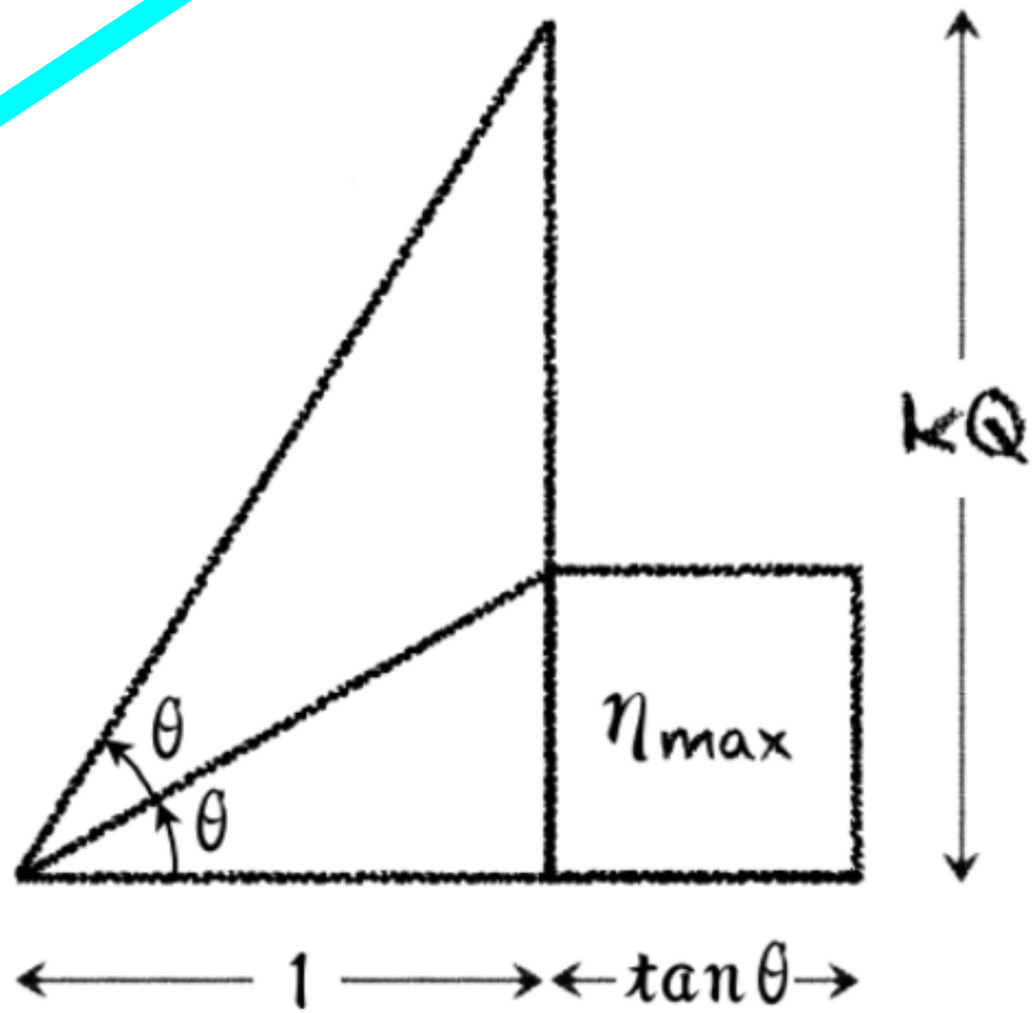
$$\mathbf{Z} = 50(\mathbf{I}+\mathbf{S})(\mathbf{I}-\mathbf{S})^{-1}$$

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \underset{\mathbf{R}}{\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}} + j\underset{\mathbf{X}}{\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}}$$

$$kQ = \frac{|Z_{21}|}{\sqrt{|\mathbf{R}|}}$$

$$kQ = \tan 2\theta$$

$$\eta_{max} = \tan^2\theta$$



Answers to the Basic Questions on *kQ*

Q1 Does *kQ* exist in every kind of coupling?

A Yes it does.



Q2 Can we derive *kQ* from circuit parameters?

A Yes we can.

$$kQ = \frac{|Z_{21}|}{\sqrt{|\mathbf{R}|}}$$

Q3 How does *kQ* relate to maximum efficiency?

A Quite in elegance.

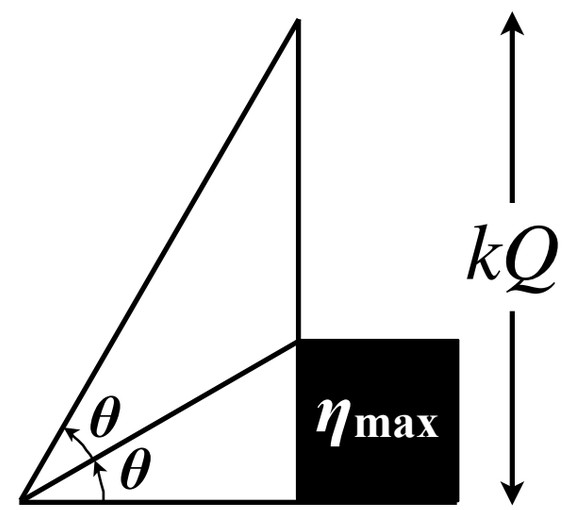
$$kQ = \tan 2\theta, \quad \eta_{\max} = \tan^2 \theta$$

Conclusion

- ***kQ*** **is an essential WPT performance index**
- **General *kQ* formula in 2-port matrix Z**
- ***kQ*** **dominates** η_{max} **by way of** θ



$$\mathbf{Z} = 50(\mathbf{I}+\mathbf{S})(\mathbf{I}-\mathbf{S})^{-1} = \mathbf{R} + j\mathbf{X}$$



$$\Rightarrow \underline{kQ} = \frac{|z_{21}|}{\sqrt{|\mathbf{R}|}} \Rightarrow \eta_{max} = \underline{\tan^2\theta}$$

Exploit *kQ* for future WPT development and evolution